Math 13 – Liberal Arts Math – Chapters 16&17 Test Solutions

1. Consider the Avis rental car number 8160040.

a. Show how to divide this number by 7
SOLN:

```
   1165720
7 | 8160040
  -7
   11
   -7
    46
   -42
     40
    -35
      50
     -49
       14
      -14
        0
```

b. If the third and fourth digits (6 and 0) of the number are exchanged, can the check digit detect the error? Explain.
SOLN Yes since 8106040 is not divisible by 7, the error would be detected. The difference between 8160040 and 8106040 is 54000, which is not a multiple of 7.

c. The difference between 8160040 and 1860040 is 630000, which is a multiple of 7, thus the error transposition of the first two digits would go undetected.

2. Is 0-2370-2990-X a viable ISBN number? If not, can the second digit be changed to produce a viable ISBN number?

SOLN: Here $\sum_{i=1}^{10}(11-i)a_i = 9*2+8*3+7*7+5*2+4*9+3*9+10 = 174 = 11*15+9$, so it's not a valid ISBN number. If the second digit were *d* instead of 2, the check sum would be $9d+8*3+7*7+5*2+4*9+3*9+10 = 174 = 9d+156 = (9d+2)+11*14$ This means we want $9d + 2$ to be a multiple of 11. $d = 1$ is the only numeral that would work.

3. Is 

a valid UPC number? Explain why or why not.
Recall that in the UPC system, the check digit sum involves multiplying the first and every other odd position digit by 3.
SOLN: 3*7+1+3*2+3+3*4+5+3*6+7+3*8+9+4=110 is a multiple of 10 so this is a valid UPC number.

4. What if the US Post Office received a letter with postnet code 19293-1234-6 and the postnet characters as shown below (separated here by spaces.)

a. From the postnet bar codes, which digit is in error?
SOLN: The fourth representation has 3 long bars when it should only have two.
b. How could the correct digit be determined automatically?
SOLN: The sum of the other digits is 31, so the error would be automatically corrected to 9 = 9 making the sum divisible by 10.

5. You propose a code in which each three-digit binary message word $a_1\ a_2\ a_3$ has appended a parity check digit $c_1 = a_1 + a_2 + a_3$.
a. Write out all the codewords.
SOLN: {0000, 0011, 0101, 1001, 0110, 1010, 1100, 1111}
b. What is the weight of the code?
SOLN: The weight of the code is 2.
c. How many errors can it detect?
SOLN: It will detect 1 error.

d. How many errors can it correct?
SOLN: It will correct 0 errors.

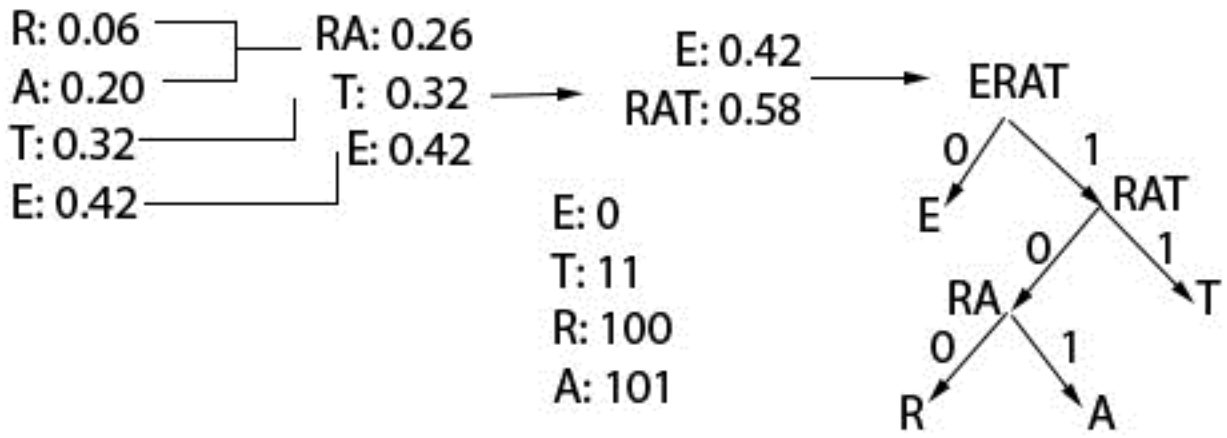
e. Could you decode 1101 with the nearest neighbor method? Explain.
SOLN: No, there is no unique nearest neighbor since both 1111 and 1001 are 1 apart from 1101.

6. Decipher the ciphertext MOMXOX using the Vigenére method with the keyword DOG.

```
SOLN:    12 14 12 23 14 23
       + 23 12 21 23 12 20
         09 00 07 20 00 17
          J  A  G  U  A  R
```

7. Suppose someone using the RSA encryption method and chooses primes $p = 3$ and $q = 17$. Find appropriate values for the other parameters of the code: n, m, r and s.
SOLN: There are lots of possibilities for r. I'll cover a few here. In every case. n = 51, $m = 16$.
If $r = 3$ then $3^3 \bmod 16 = 11$ so $3^4 \bmod 16 = 33 \bmod 16 = 1$. Thus $s = 11$.
If $r = 5$ then $5^2 \bmod 16 = 9$ so $5^3 \bmod 16 = 45 \bmod 16 = 13$ so $5^4 \bmod 16 = 65 \bmod 16 = 1$. Thus $s = 13$.
If $r = 7$, then $7^2 \bmod 16 = 1$ so $s = 7$.

8. Assume that a dataset of four letters occurs with the following probabilities:
E: 0.42, T: 0.32, A: 0.20, R: 0.06
Use a Huffman tree to create a Huffman code for E, T, A and R.



9. Use a truth table to show that $P \wedge (Q \vee R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$.

$p \wedge (q \vee r) \quad (p \wedge q) \vee (p \wedge r).$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F